Why voting?

A welfare analysis

Moritz Drexl and Andreas Kleiner

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Introduction

Textbook example:
A group of privately informed agents decides whether or not to build a bridge.

- Voting is criticized for being inefficient
- The efficient decision rule can be implemented by VCG mechanisms, but not with a balanced budget

Question

Which decision rule maximizes expected welfare of the agents?

Outline

- Literature
- Model
- Results
- Discussion

Literature

- **Public decision making with monetary transfers**
  - Efficient decision rule induces budget imbalances (Green and Laffont, 1979)
  - Suggestion to use the Pivot mechanism to maximize welfare (Tideman and Tullock, 1976).
- **Decision making without money**
  - Optimal voting rules (Rae, 1969; Schmitz and Tröger, 2012)
  - Decision rules based on wasteful signaling (Hartline and Roughgarden, 2008; McAfee and McMillan, 1992)
- **Justification for voting**
  - Ledyard and Palfrey (2002)
  - Bierbrauer and Hellwig (2012)

Model: Set-up

- N agents
- decide whether to accept ($X = 1$) or reject ($X = 0$) a given costless proposal.
- Agent i values proposal with θ_i, which is observed privately.
- Utility: $\theta_i X + T_i$
- Type space $\Theta = [\underline{\theta}, \ \overline{\theta}]$, with $\underline{\theta} < 0 < \overline{\theta}$
- Valuations are drawn according to a distribution function F, which admits a strictly positive density f and is symmetric across agents.

Model: Definitions

Definition

- A **social choice function (scf)** is a tuple (x, t) such that

$$x : \Theta^N \to \{0, 1\},$$
$$t : \Theta^N \to \mathbb{R}^N.$$

- (x, t) is **feasible** if, for all θ, $\sum_i t_i(\theta) \leq 0$.
- (x, t) is **strategy-proof** if truthful reporting is a dominant strategy.
- (x, t) satisfies **universal participation** if, for all i and θ,

$$\theta_i x(\theta) + t_i(\theta) \geq \theta_i \underline{x}_i(\theta_{-i}).$$

- (x, t) is **anonymous** if, for all θ, $x(\theta) = x(\hat{\pi}(\theta))$.

Model: Objective function

Expected utilitarian welfare under scf (x, t):

$$U(x,t) := \mathbb{E}_\theta \left[\sum_i \theta_i x(\theta) + t_i(\theta) \right]$$

Comments:

- Expectation with respect to prior distribution
- Utilitarian welfare, takes payments into account
- Inclusion of payments would not matter if we only imposed ex-ante feasibility or Bayesian incentive compatibility

Results: Characterization of incentive compatibility

Lemma

A scf (x, t) is strategy-proof if and only if, for each agent i,

1. $x(\theta_i, \theta_{-i})$ *is nondecreasing in* θ_i *for all* θ_{-i} *and*
2. *there exists a function* $h_i(\theta_{-i})$*, such that for all* θ*,*

$$t_i(\theta) = \underbrace{-\theta_i x(\theta) + \int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta}_{} + \underbrace{h_i(\theta_{-i})}_{}.$$

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Rewrite objective function:

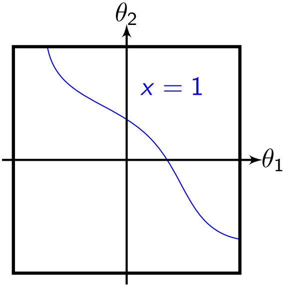
$$U(x,t) = \mathbb{E}_\theta\left[\sum_i \theta_i x(\theta) + t_i(\theta)\right]$$
$$= \mathbb{E}_\theta\left[\sum_i \int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta + h_i(\theta_{-i})\right]$$

Results: Fixing redistribution payments

Lemma

Let (x, t) be a feasible and anonymous scf satisfying universal participation. Then $h_i(\theta_{-i}) = 0$ for all i and θ_{-i}.

Results: Fixing redistribution payments

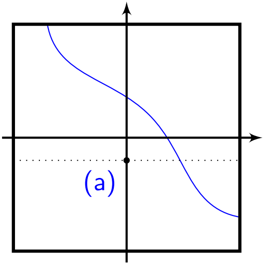


Definition

Agent i is pivotal at profile θ if $x(\theta) \neq x(0, \theta_{-i})$.

$\Rightarrow$ incentive payments of non-pivotal agents are 0

Results: Fixing redistribution payments



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Proof.

- *Step 1: For all θ_{-i}, there exists θ_i such that no one is pivotal at (θ_i, θ_{-i}).*
  - (a) Either $(0, \theta_{-i})$ satisfies the claim,
  - (b) or $(\theta_{j^*}, \theta_{-i})$, where j^* is the agent sending the highest report.

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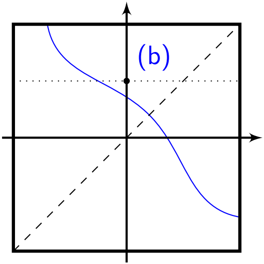

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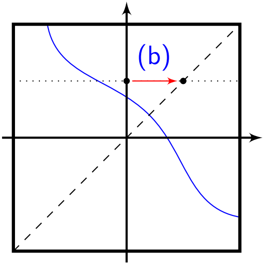

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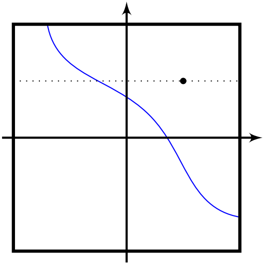

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Proof.

- *Step 1: For all θ_{-i}, there exists θ_i such that no one is pivotal at (θ_i, θ_{-i}).*
  (a) Either $(0, \theta_{-i})$ satisfies the claim,
  (b) or $(\theta_{j^*}, \theta_{-i})$, where j^* is the agent sending the highest report.
- *Step 2: $h_i(\theta_{-i}) = 0$ for all i and θ_{-i}.*
  - Participation constraint implies $h_i(\theta_{-i}) \geq 0$.
  - $h_i(\theta_{-i}) > 0$ would contradict feasibility.

Results: Fixing redistribution payments

Lemma

Let (x, t) be a feasible and anonymous scf satisfying universal participation. Then $h_i(\theta_{-i}) = 0$ for all i and θ_{-i}.

Corollary

An anonymous scf is implementable with a balanced budget if and only if it is implementable by qualified majority voting.

Results: Deriving the optimal scf.

Rewrite the objective function:

$$U(x, t) = \int \sum_i \left[\theta_i x(\theta) + t_i(\theta)\right] dF(\theta)$$

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&= \int \sum_i \left[\int_0^{\theta_i} x(\beta, \theta_{-i}) d\beta + h_i(\theta_{-i}) \right] dF(\theta) \\
&= \int \left[\sum_i \psi_i(\theta) \right] x(\theta) dF(\theta),
\end{aligned}
$$

where

$$
\psi_i(\theta) = \begin{cases} \frac{-F(\theta_i|\theta_{-i})}{f(\theta_i|\theta_{-i})} & \text{if } \theta_i \leq 0, \\ \frac{1-F(\theta_i|\theta_{-i})}{f(\theta_i|\theta_{-i})} & \text{otherwise.} \end{cases}
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Proposition

The welfare-maximizing scf maximizes $\mathbb{E}\left[\sum_i \psi_i(\theta)x(\theta)\right]$ *subject to* x *being pointwise non-decreasing.*

Results: Independent and regular distributions

$$\psi_i(\theta) = \begin{cases} \frac{-F(\theta_i)}{f(\theta_i)} & \text{if } \theta_i \leq 0, \\ \frac{1-F(\theta_i)}{f(\theta_i)} & \text{otherwise.} \end{cases}$$

- With independent types and a standard regularity condition, $\sum_i \psi_i(\theta)$ is decreasing within each orthant.
- IC implies that $x(\theta)$ is increasing in each component.
- Therefore, it is optimal to set $x(\theta)$ constant in this orthant.
- The optimal scf conditions only on the number of agents who are in favor.

Results: Optimality of qualified majority voting

Definition

A scf is called **qualified majority voting** with threshold m if $t \equiv 0$ and $x(\theta) = 1$ if and only if $|\{i : \theta_i \geq 0\}| \geq m$.

Proposition

Suppose types are drawn independently, $\frac{f(\cdot)}{1-F(\cdot)}$ *is increasing for* $\theta_i > 0$ *and* $\frac{f(\cdot)}{F(\cdot)}$ *is decreasing for* $\theta_i < 0$.

Then the welfare-maximizing scf does not involve monetary transfers and is implementable by qualified majority voting.

Results: Optimality of qualified majority voting

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A scf is called **qualified majority voting** with threshold m if $t \equiv 0$ and $x(\theta) = 1$ if and only if $|\{i : \theta_i \geq 0\}| \geq m$.

Proposition

Suppose types are negatively affiliated, $\frac{f(\cdot|\theta_{-i})}{1-F(\cdot|\theta_{-i})}$ *is increasing for* $\theta_i > 0$ *and* $\frac{f(\cdot|\theta_{-i})}{F(\cdot|\theta_{-i})}$ *is decreasing for* $\theta_i < 0$.

Then the welfare-maximizing scf does not involve monetary transfers and is implementable by qualified majority voting.

Results: Irregular distributions

Proposition

Suppose types are drawn independently. Then the welfare-maximizing scf is such that $x(\theta) = 1$ *if and only if* $\sum_i \overline{\psi}(\theta_i) \geq 0$ *where* $\overline{\psi}$ *denotes the (Myerson-)ironed* ψ.

For dependent types, we do not have an explicit description for the ironing procedure.

Discussion

- Dropping the anonymity requirement allows to implement "sampling Groves schemes"
- Numerical evidence that results could continue to hold without participation constraints.